Quality Measures and Ratios: Assignments

Marc van Kreveld

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The lecture on measures and ratios of measures encourages the attention for geometric measures: their design and properties. Well known measures are distance measures for shapes, like Hausdorff distance, Fréchet distance (for curves), and area of overlap (for polygons). Other measures are dilation (detour factor), density, Earth mover's distance, homotopy height, root-mean-square error, ...

In graph drawing, measures are used to capture the quality of a drawing of a graph. All graphs have many different drawings, and some are better than others.

1 Designing measures

We design a new measure if there is no existing measure that captures what we want to represent. Using many small examples, we can test if the measure does what we want: whether it discriminates on the things we care about.

Assignment 1: lake adjacency measure

Design a measure that captures, for a path C, that it often runs parallel to the boundary of big lakes. Assume we have a set of polygons (lakes) given.

Check your measure for being simple, discriminating, scale-invariant, and robust. Note that in many geographic situations, scale-invariance is not an asset; it is just something we can examine about a measure.

To test for being discriminating, draw a reasonable situation and consider the score for a curve. Imagine changing the situation slightly (slight move of the curve, slight enlargement of a lake, local change that makes the curve slightly more parallel to a nearly lake), and see if the measure responds suitably.

Assignment 2: forest density variation measure

Design a measure that captures, for a polygon representing a forest, that its density of trees is varied. Assume we have a set of points (trees) inside a polygon (forest) given as the input.

Check your measure for being simple, discriminating, scale-invariant, and robust. Again, draw examples and examine your measure.

Assignment 3: t-norms and t-conorms

We want to examine possible paths through a nature area for being "good". To this end, imagine a score for being reasonable in its shape (avoiding double-backing, getting far away compared to length of path, no self-intersections, ...), which is a score in $[0, 1]$. Imagine a different score for scenic beauty (different landscapes, sweeping views, ...). For the quality of a path that takes both into account, would you use a t-norm or a t-conorm? Why?

Assuming both separate scores are "reasonable", which t-norm/t-conorm would you choose?

2 Quality ratios

One of the more interesting quality ratio topics is considering the measure angular resolution, and examining how bends can improve this measure. So we will compare free straight-line planar drawings with free 1-bend (or more bend) straight-line planar drawings.

Assignment 4: 1-bend drawings

Suppose we allow one bend in each edge, and we wish to analyze how much better the angular resolution may become when allowing a bend. We choose to let the bend itself also contribute to the angular resolution, so you don't want to make a very sharp turn in an edge.

For specific graphs like $C_3 = K_3$, C_4, and K_4, what are the quality ratio lower bounds we get for these examples?

What are the quality ratios obtained for the Platonic solids?

Can you find any planar graph giving a better lower bound on the quality ratio of 1-bend drawings, versus, free straight line drawings?

Assignment 5: 2-bend drawings

Suppose we allow an extra bend. How much can that help? Answer the questions of the previous assignment by comparing 2-bend drawings with 1-bend drawings, and 2-bend drawings with no-bend drawings.

Assignment 6: many-bend drawings

Suppose you are allowed arbitrarily many bends. What is the quality ratio for many-bend drawings when compared to no-bend drawings?

At what point doesn't it help anymore to get more bends? Is there a value (function of n, the number of vertices in the graph) so that we can always restrict ourselves to at most that many bends? You may wish to distinguish the cases of graphs with no degree-3 or more nodes, and graphs that have at least one node of degree at least 3.

Assignment *

In case your are done and feel like doing more, try other quality measures for ratios of bend drawings.

Another option is to consider any two drawing styles and see what you can get for bounded-face area/diameter ratio as the quality measure.

3 Solutions?

Making these assignments is ideally done in small groups. The assignments were composed to train with the topics. Understanding an assignment and knowing how to think to make them, or solve them, is already enough. It is for now not important to get the best possible solutions.

In many cases I don't know what the best solution is.