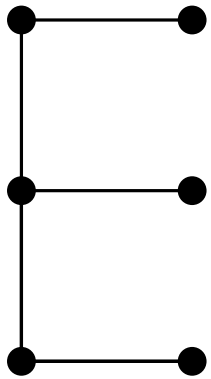
ETH zürich

Finding an Induced Subtree in an Intersection Graph is often hard

Hidefumi Hiraishi, University of Tokyo
Dejun Mao, University of Tokyo
Patrick Schnider, ETH Zürich

EuroCG 2020, Würzburg

Introduction



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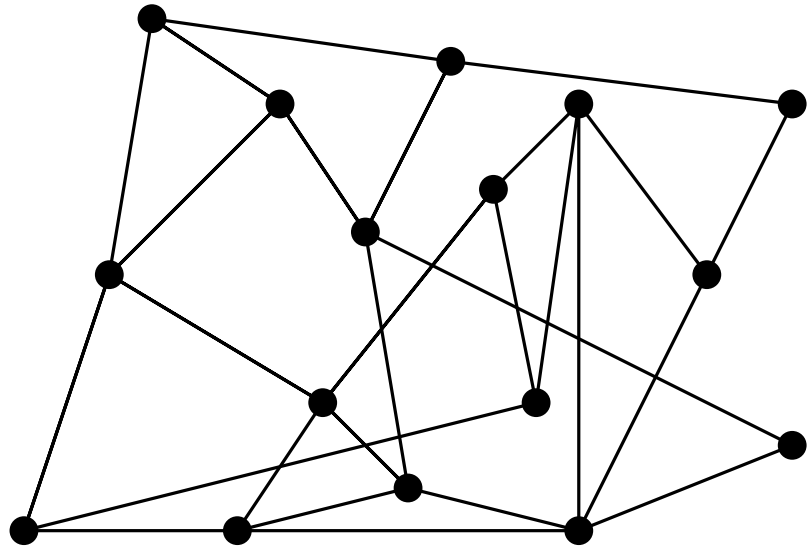

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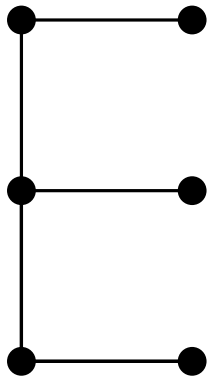

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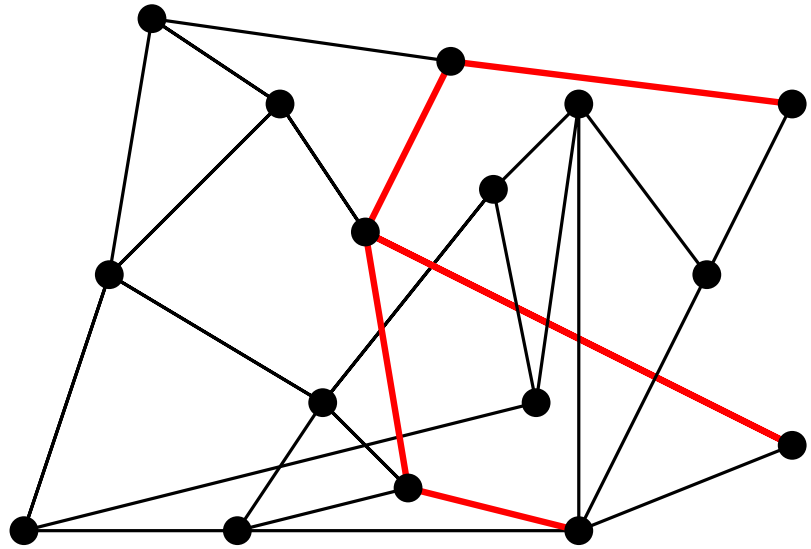

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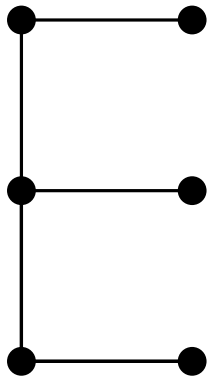

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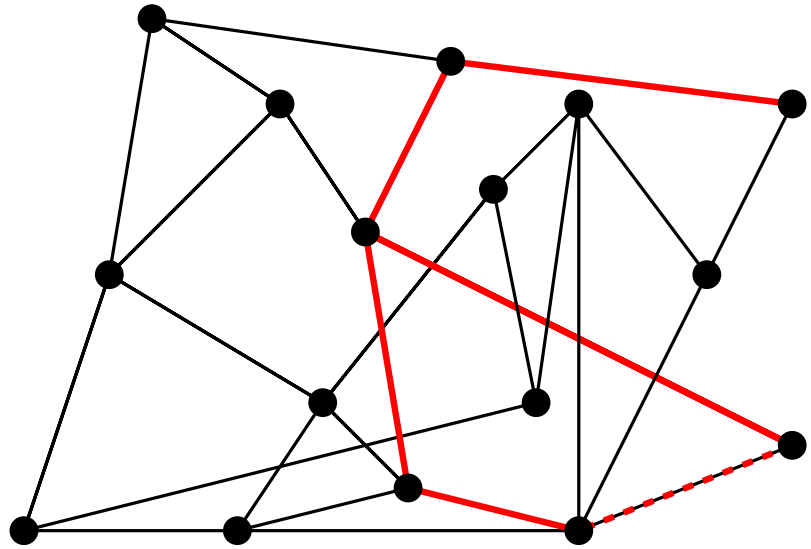

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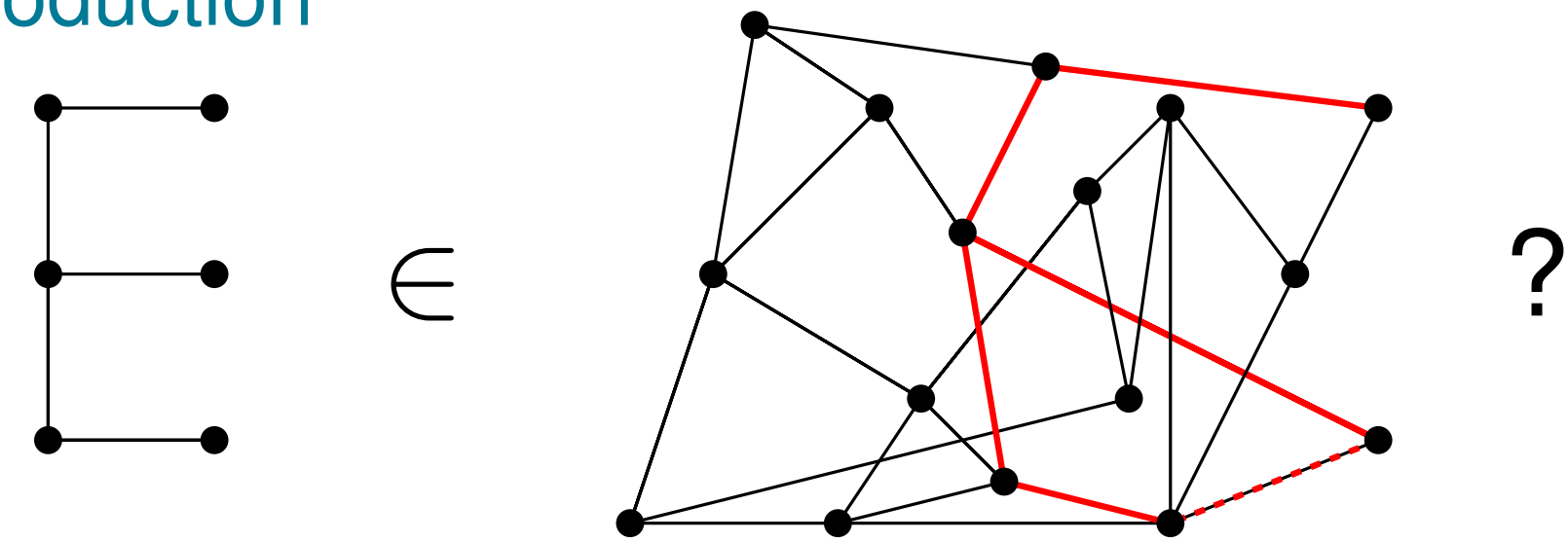

Subtree Isomorphism Problem (STI)

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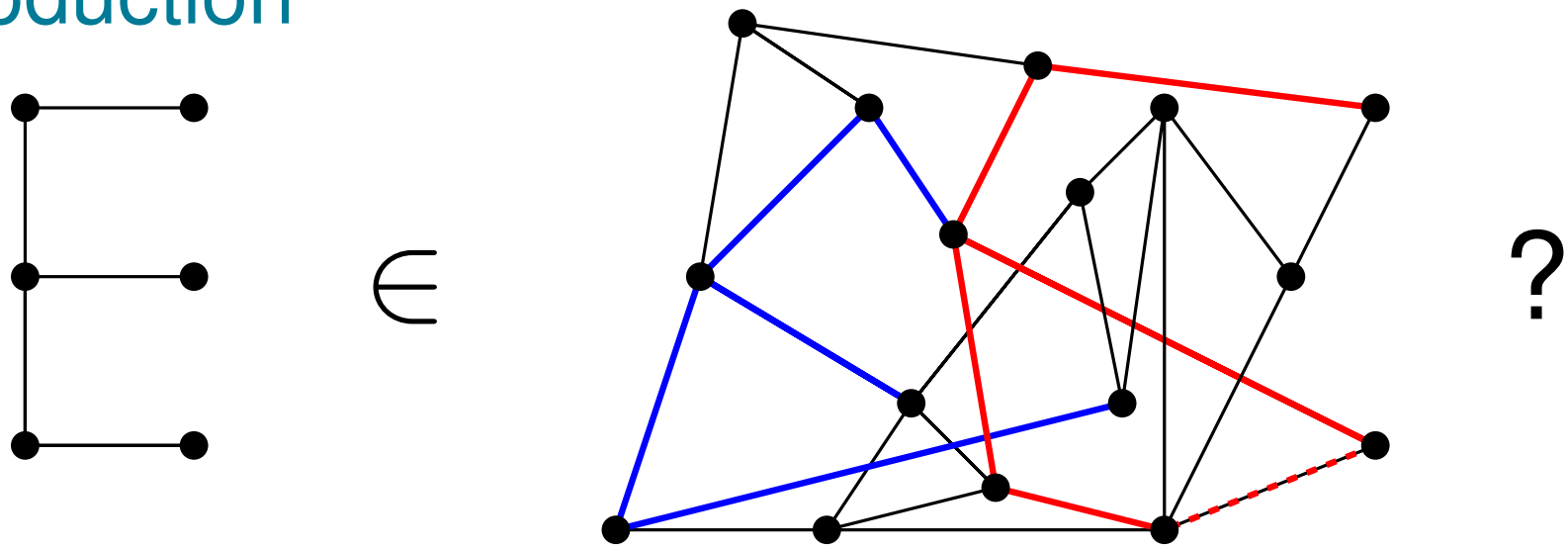

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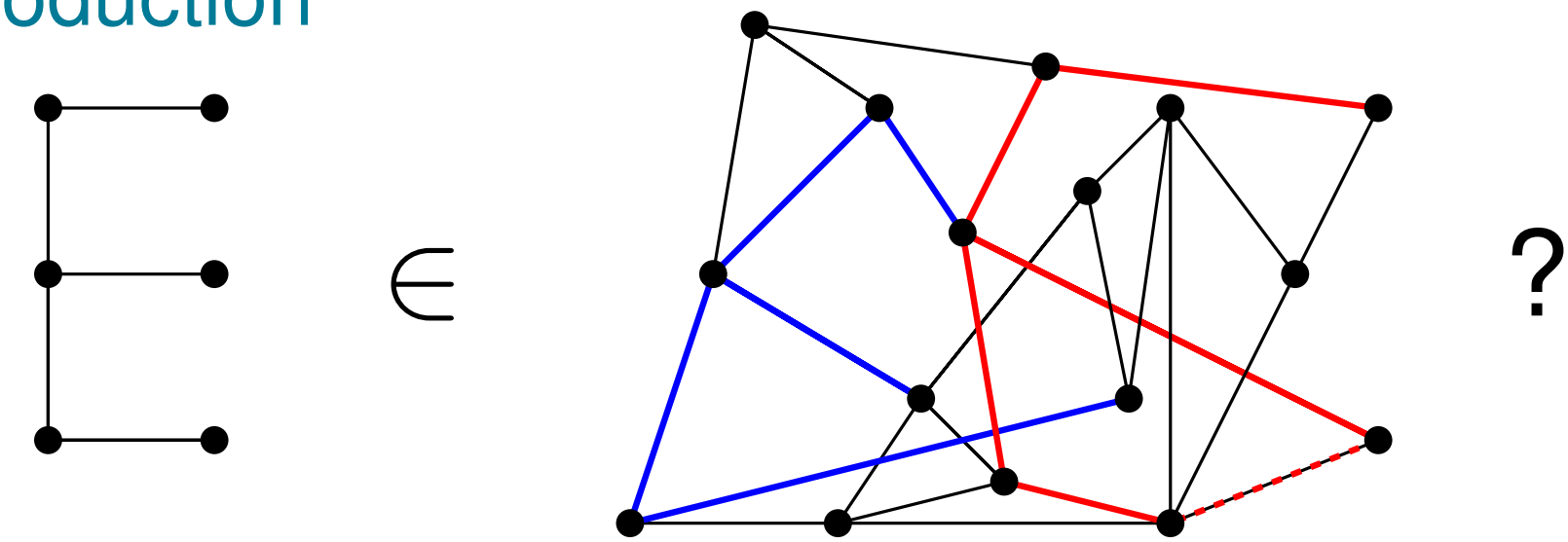

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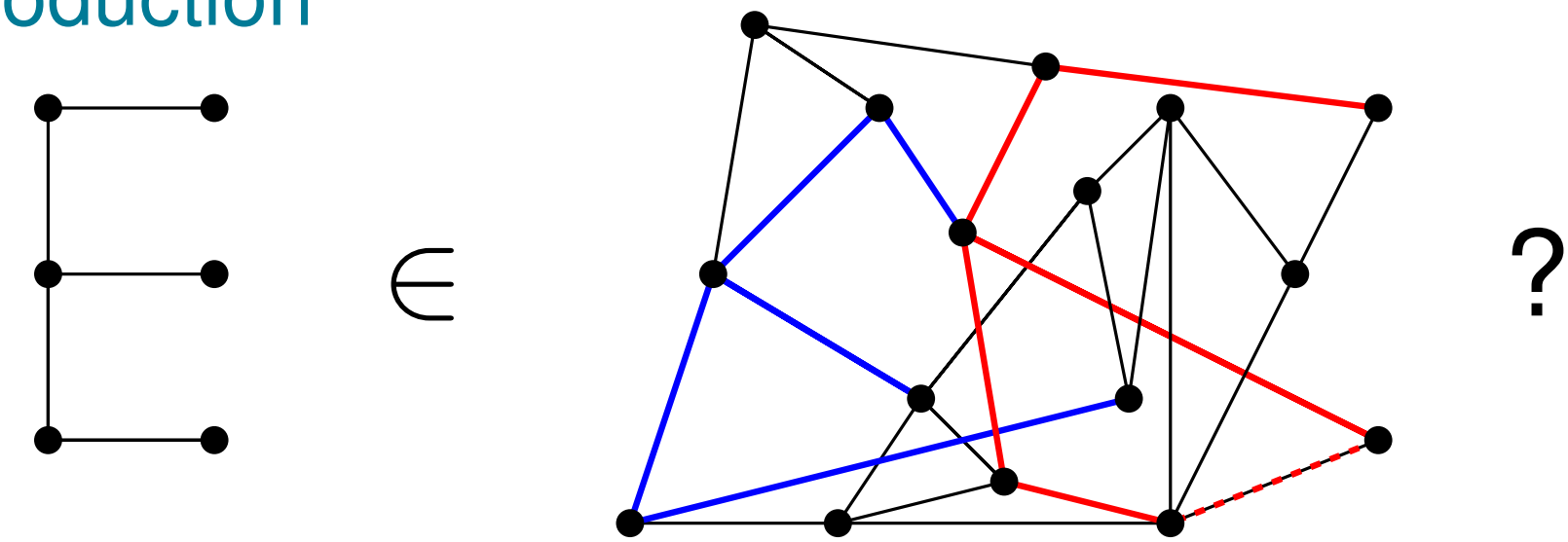

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Induced Subtree Isomorphism Problem (ISTI)

NP-complete for graph classes s.t.

-closed under subdivision of edges and

-Hamiltonian path is hard

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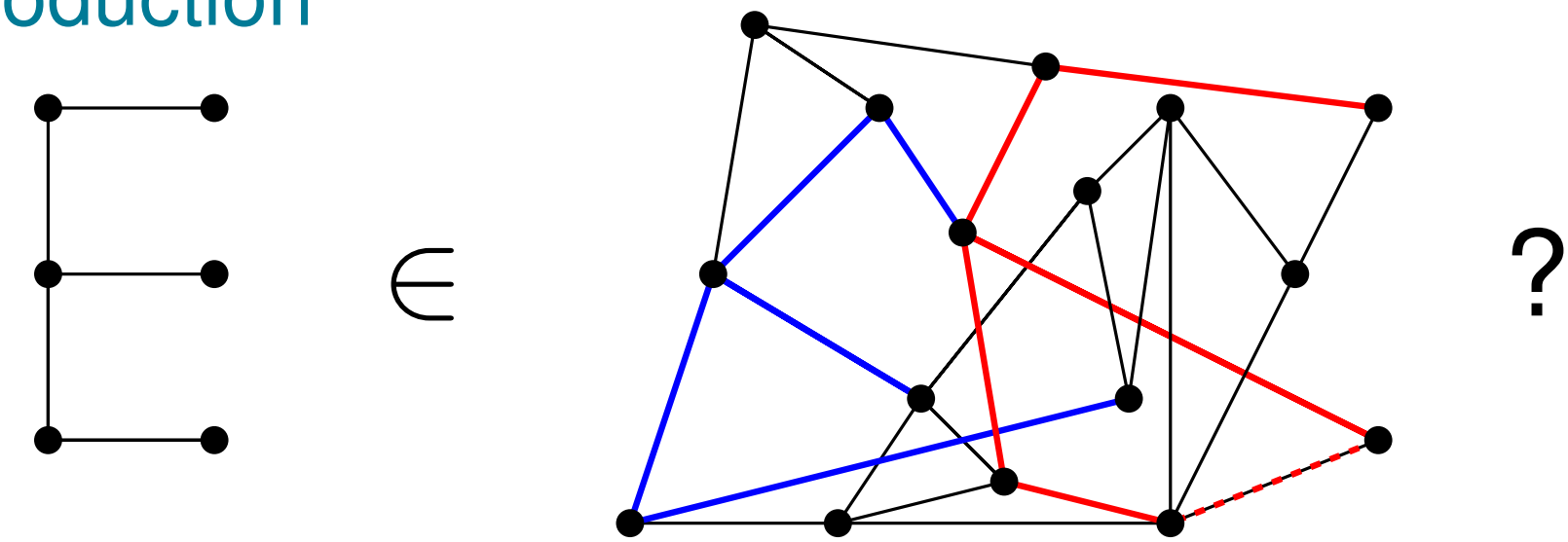

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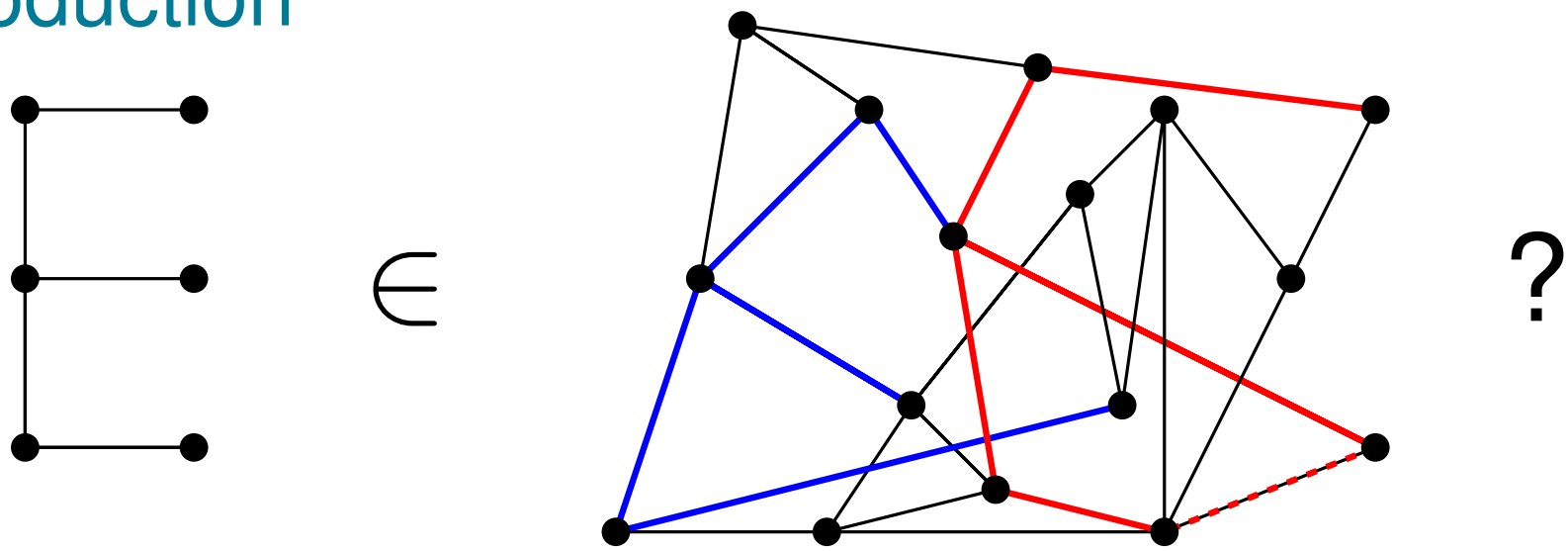

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Interval graphs: STI is hard but ISTI is poly-time

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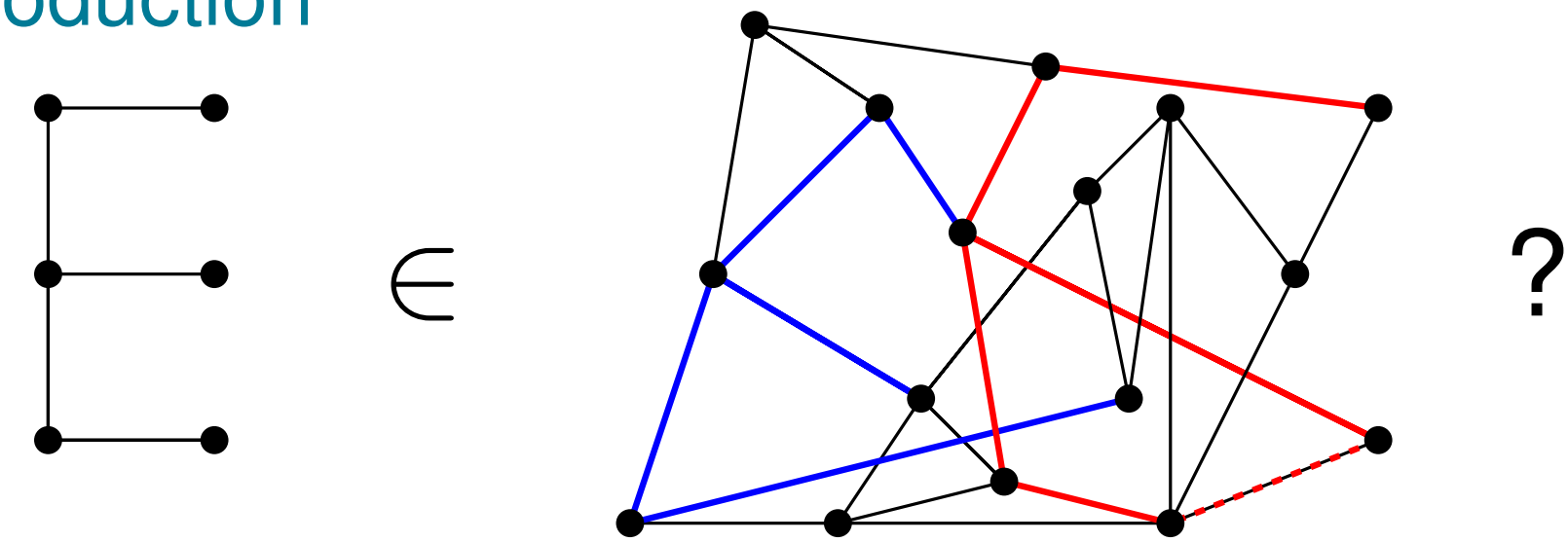

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This talk: NP-complete for many intersection graphs

The reduction for planar graphs

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$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_4)$$

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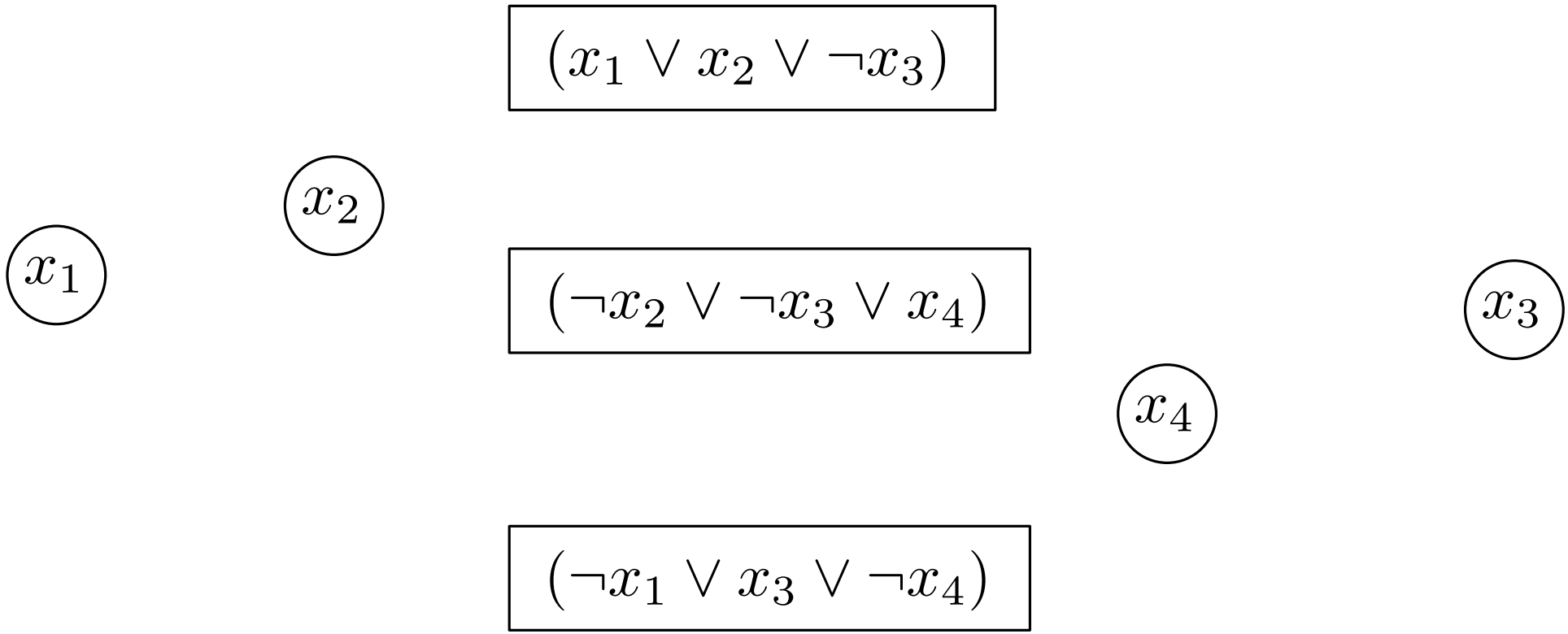

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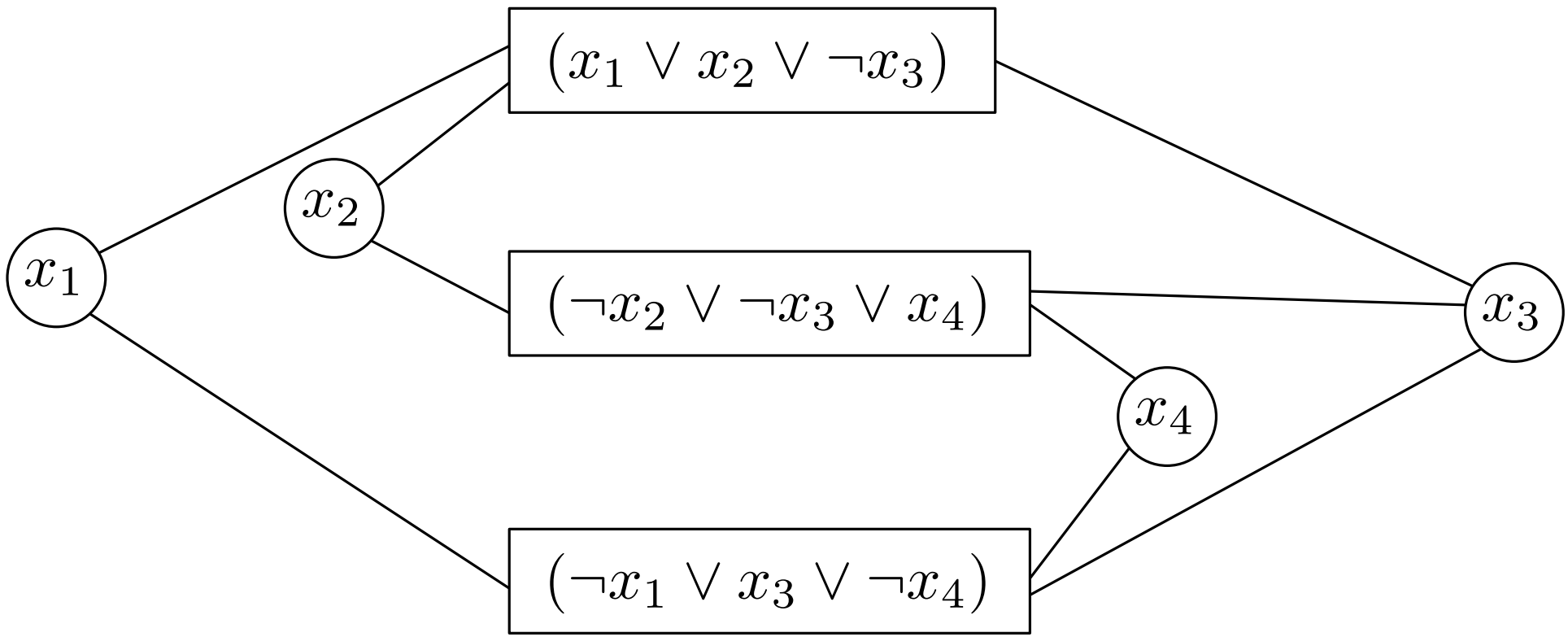

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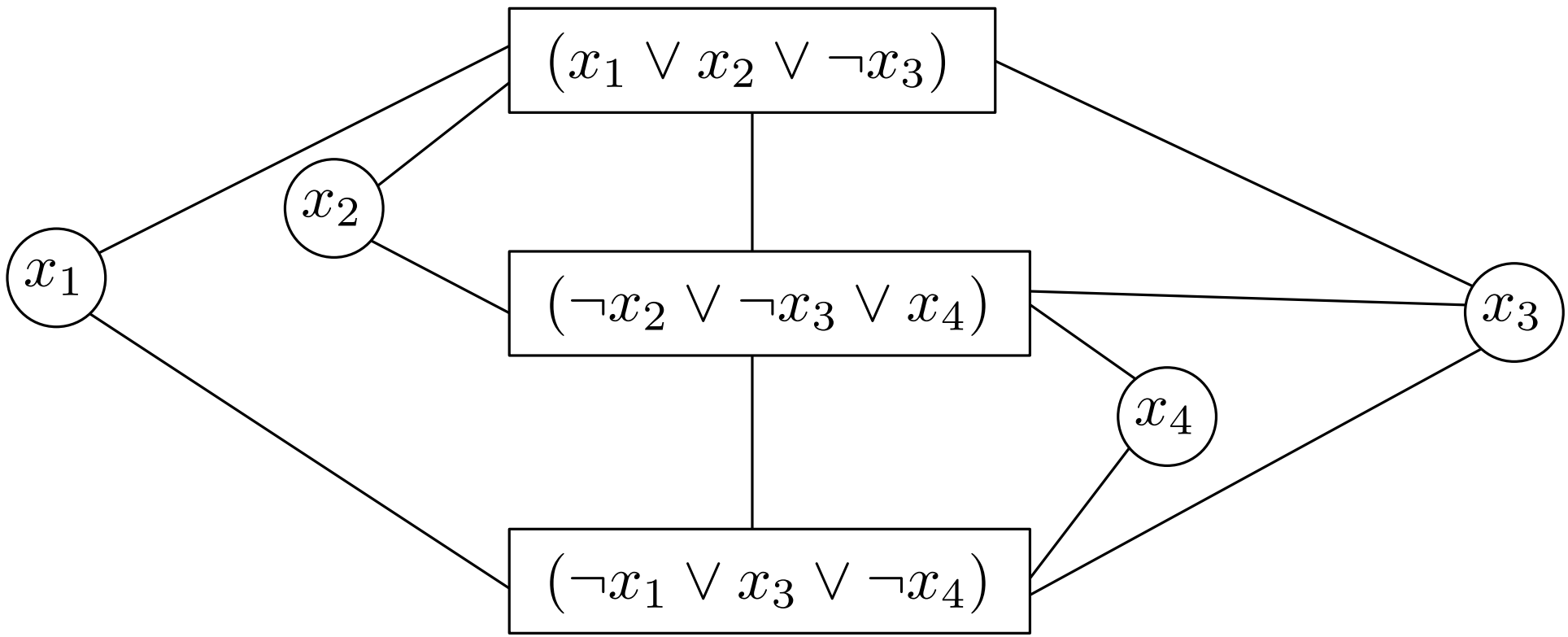

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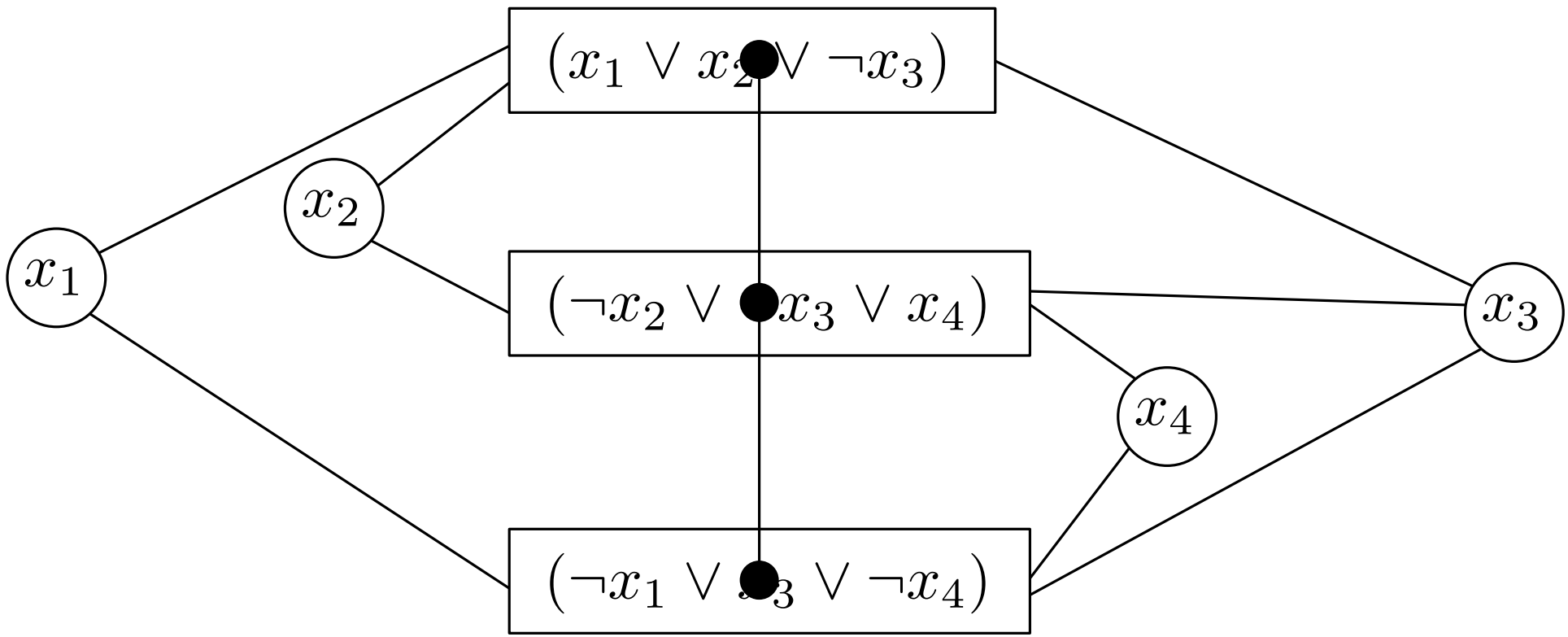

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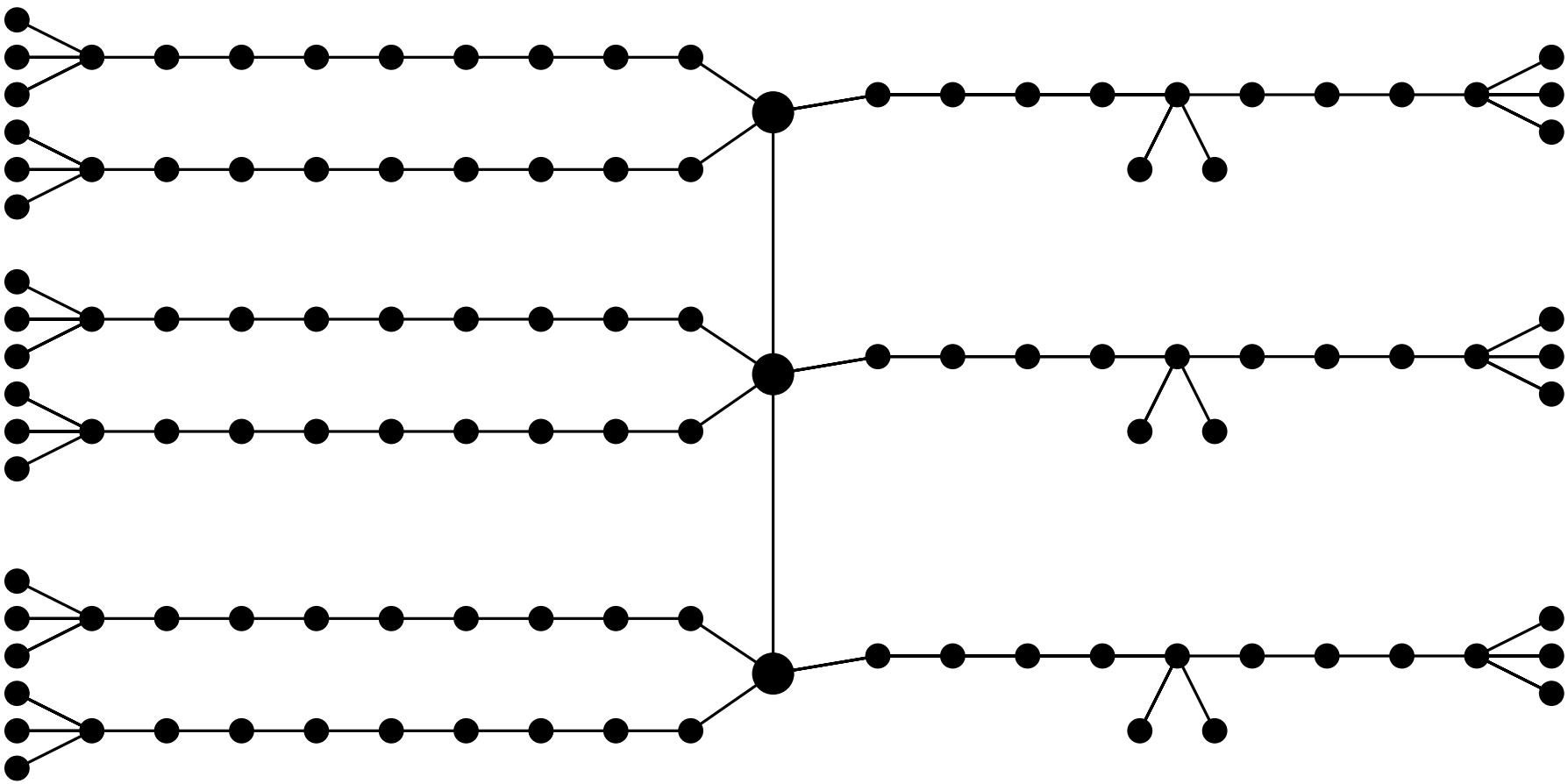

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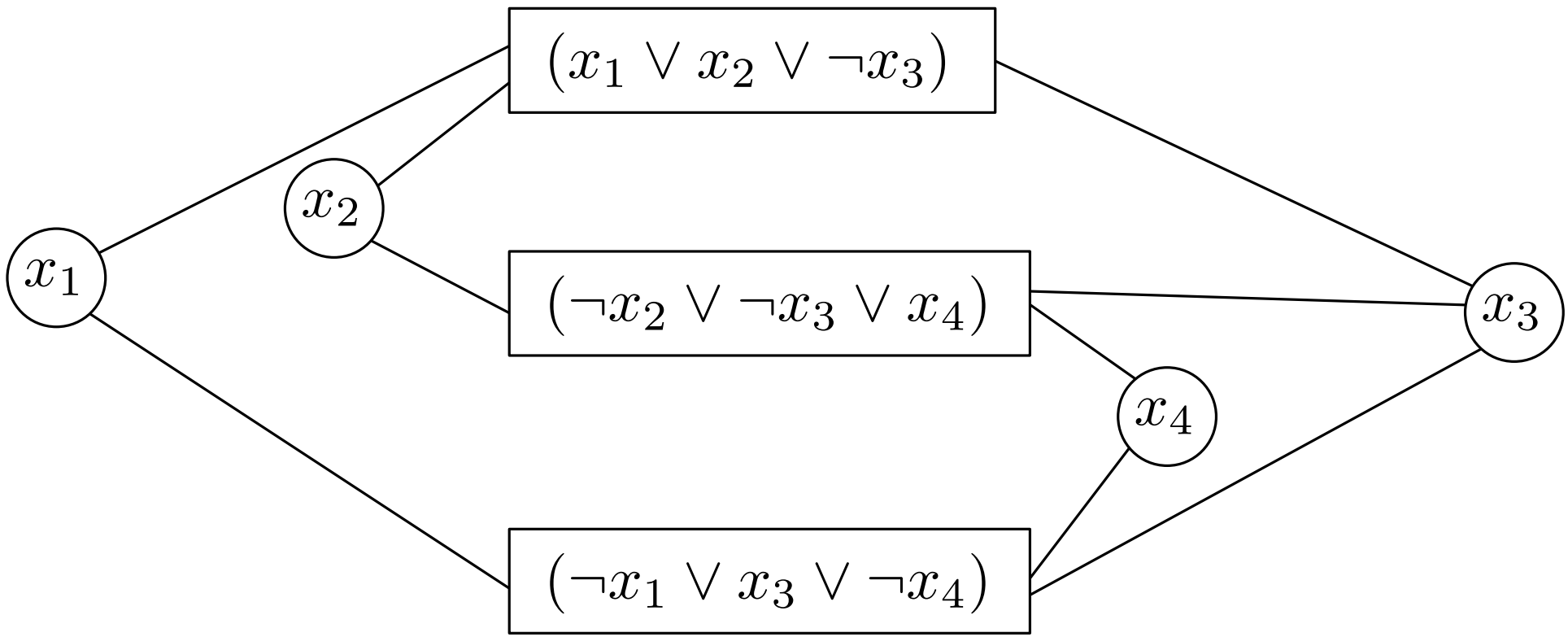

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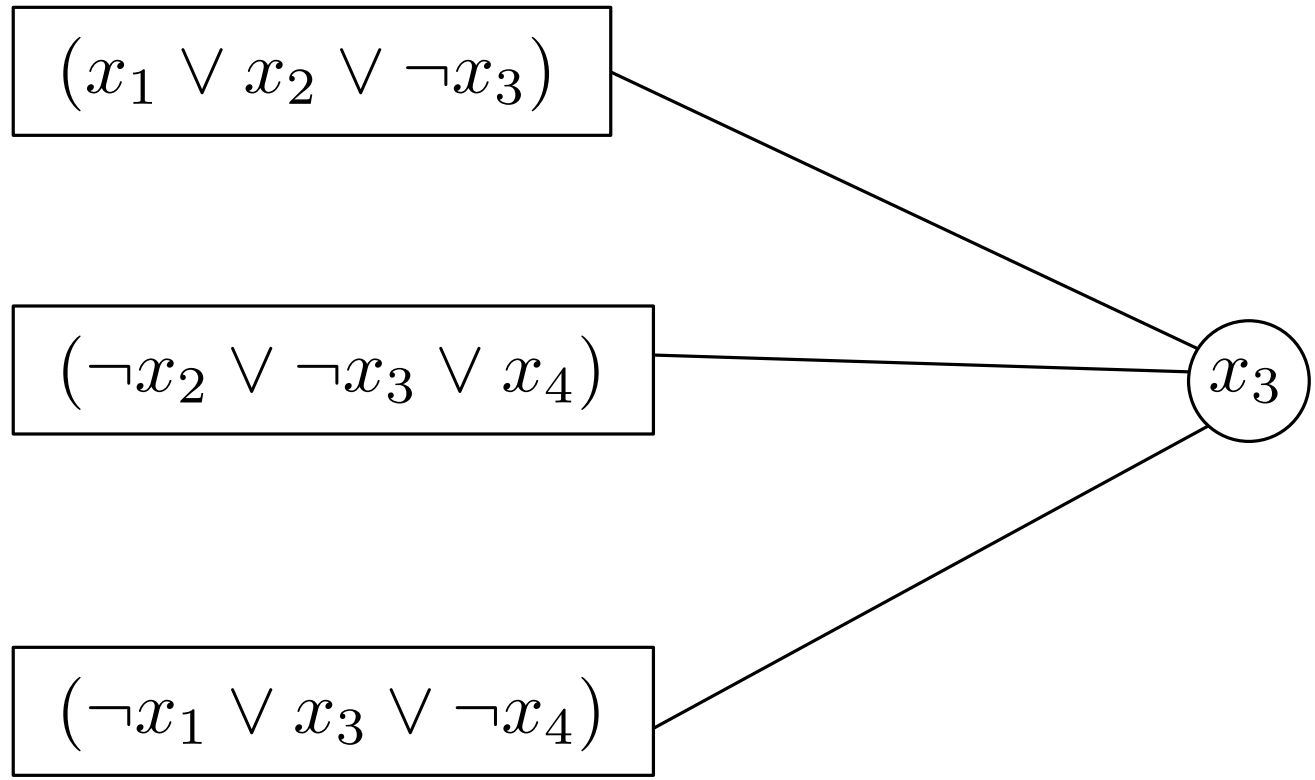

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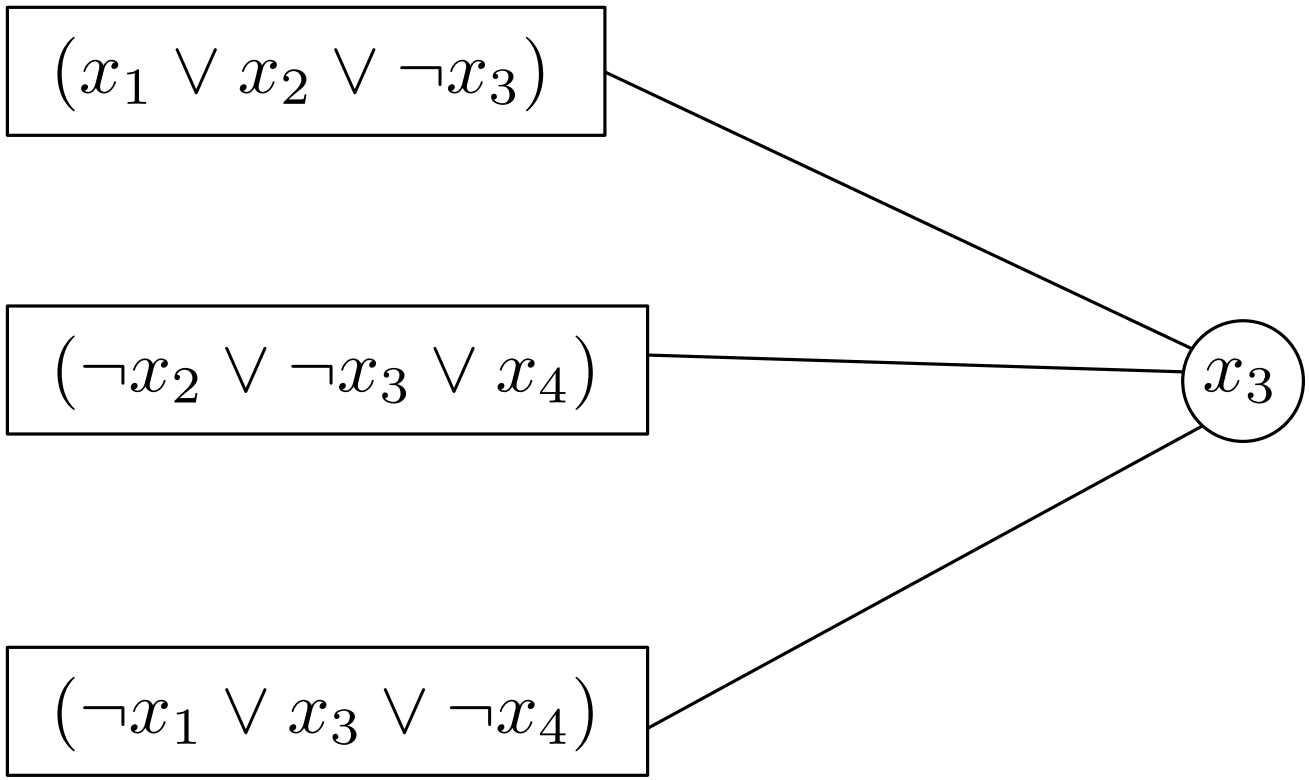

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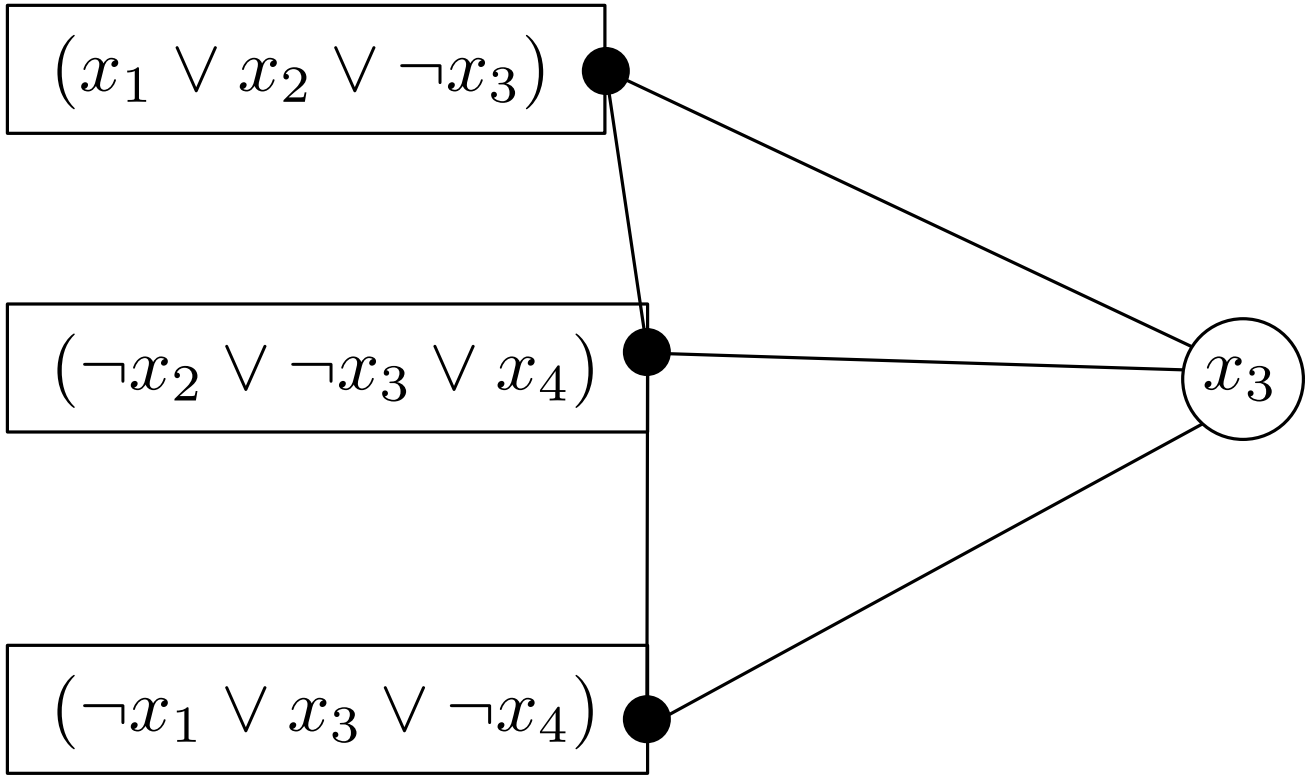

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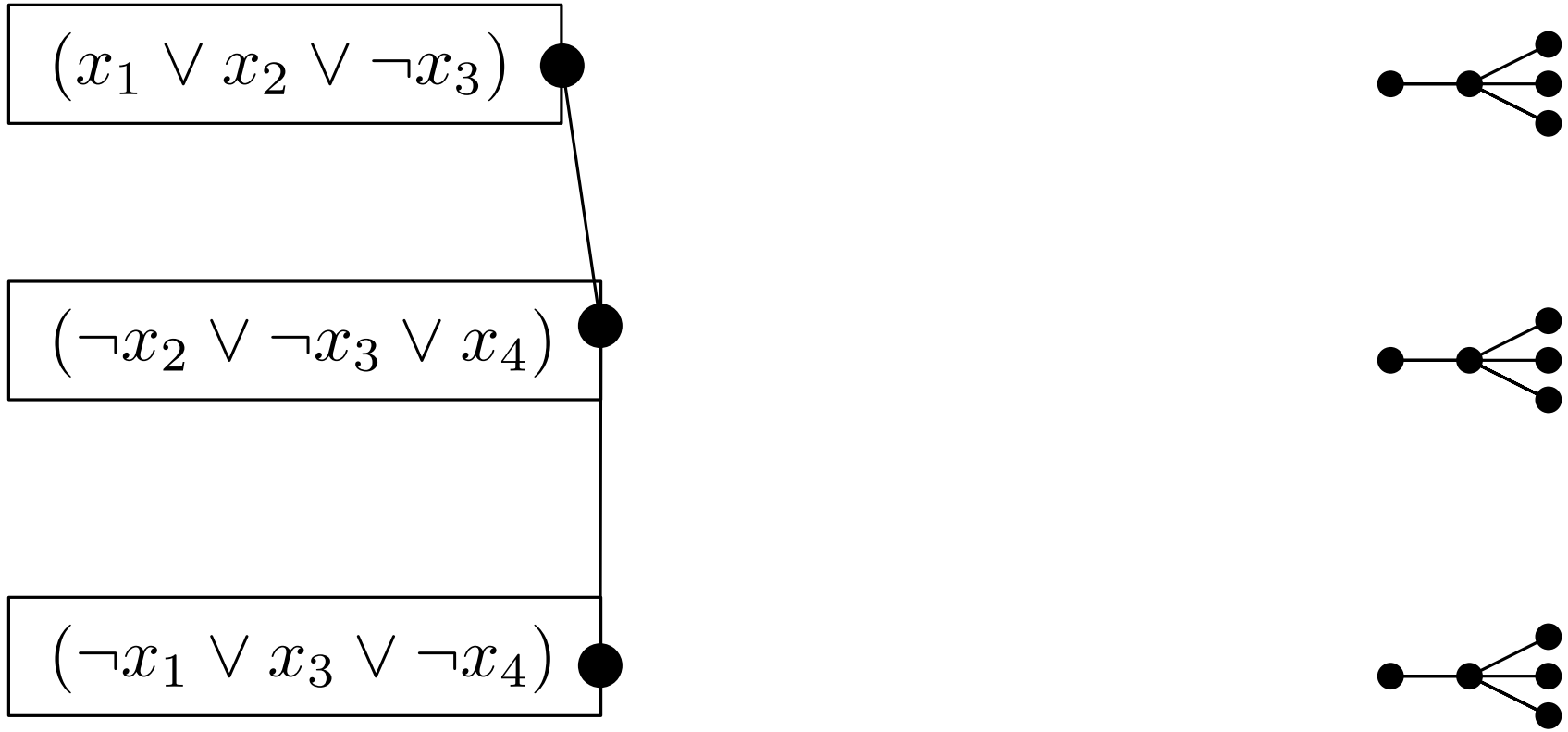

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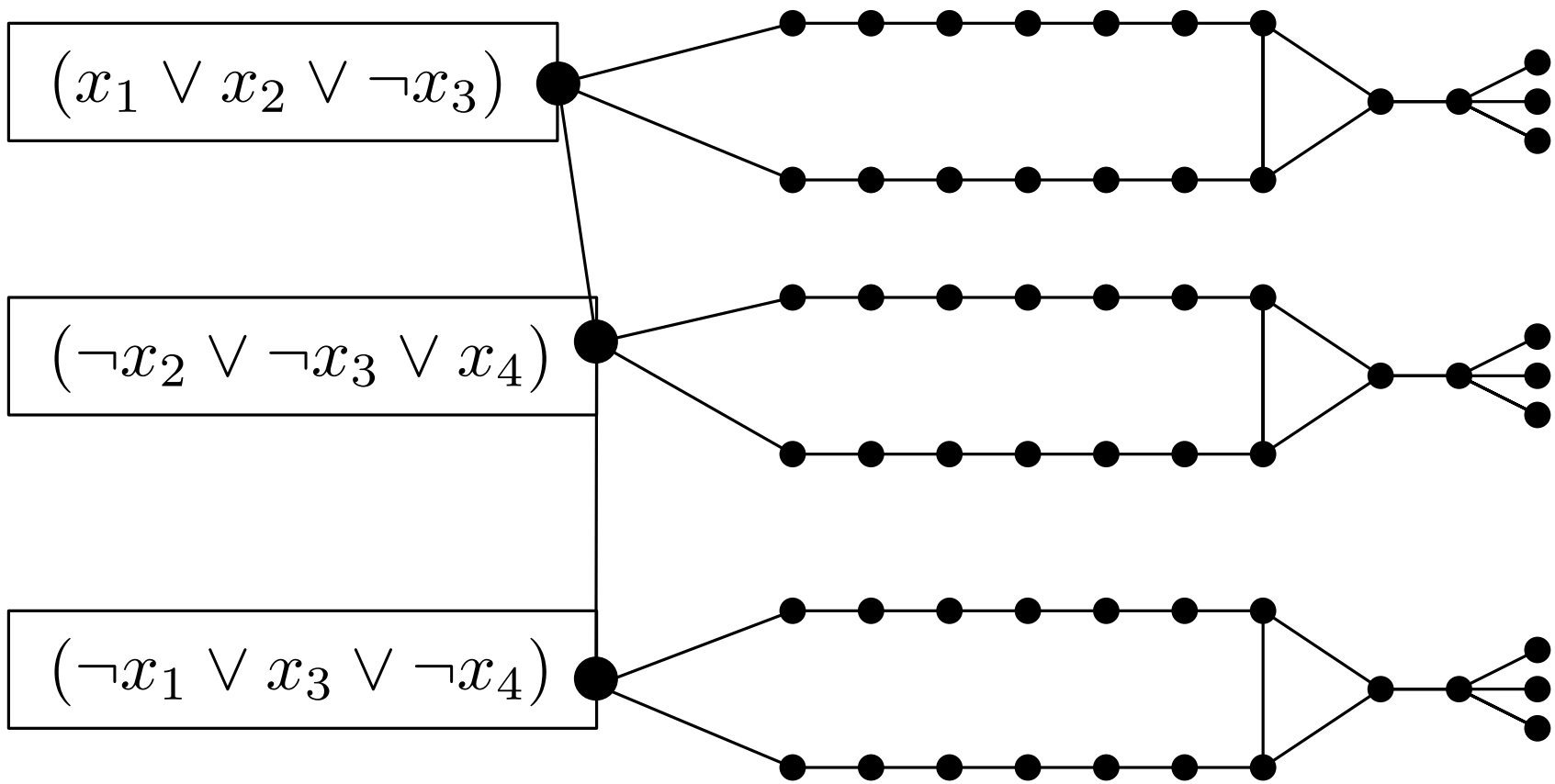

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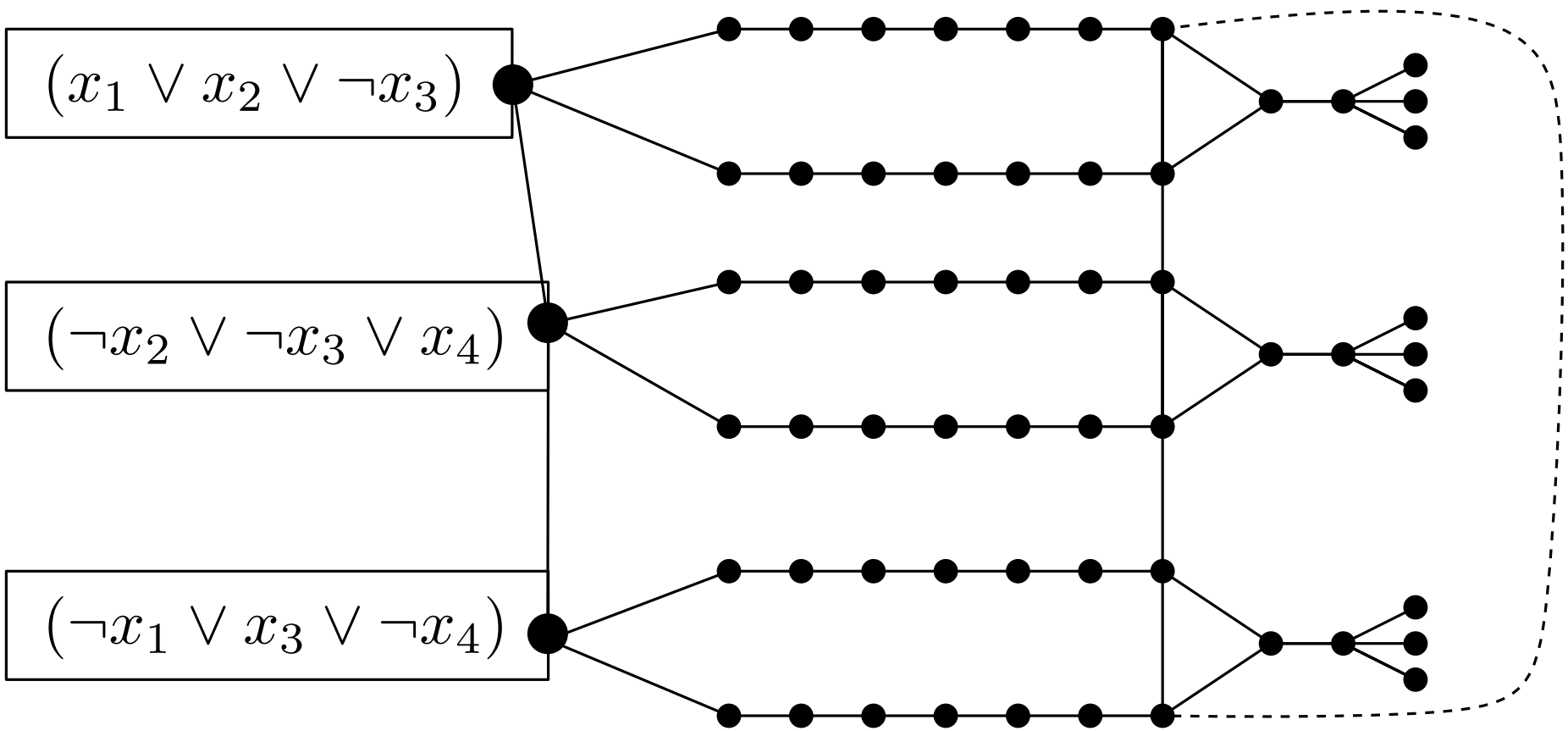

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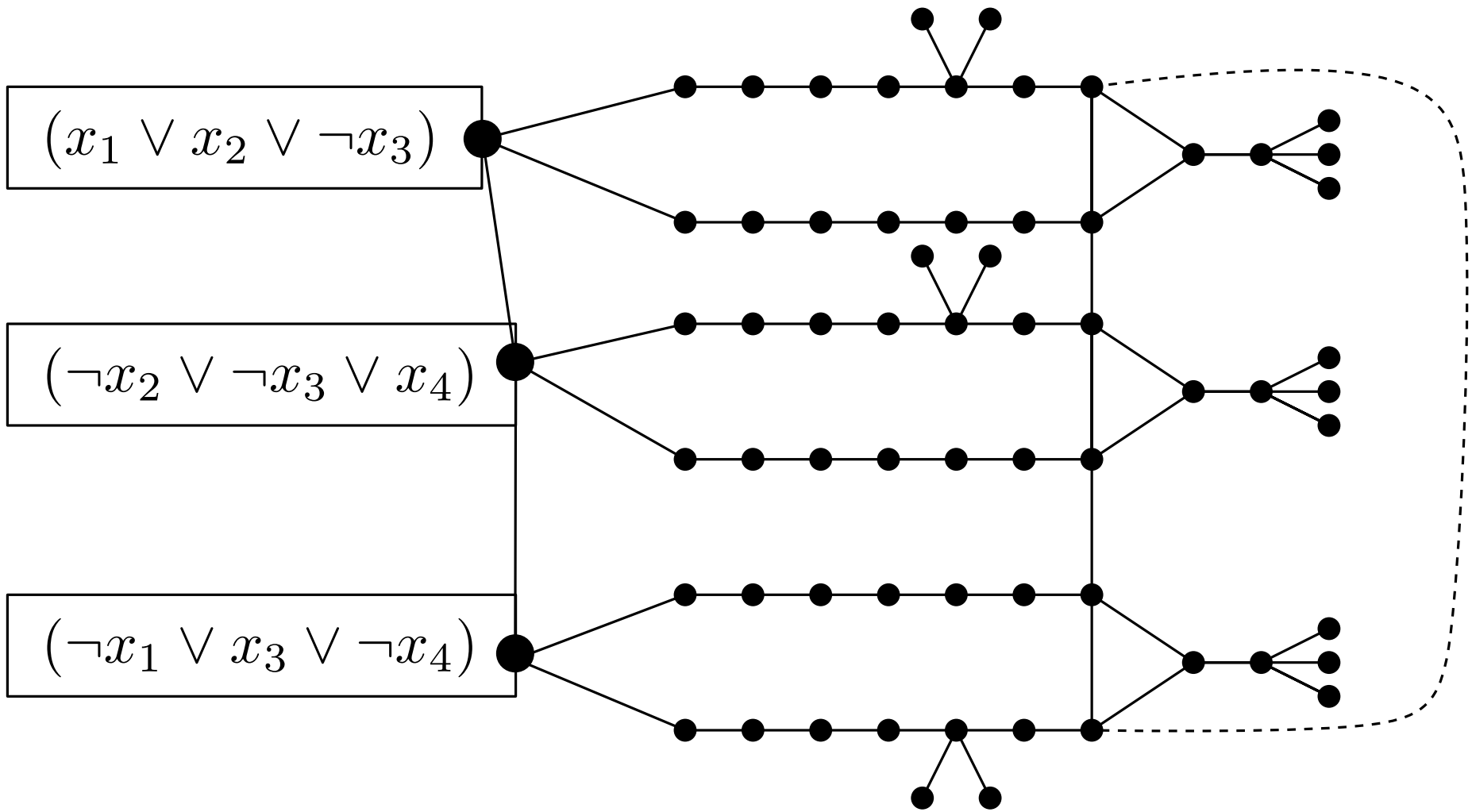

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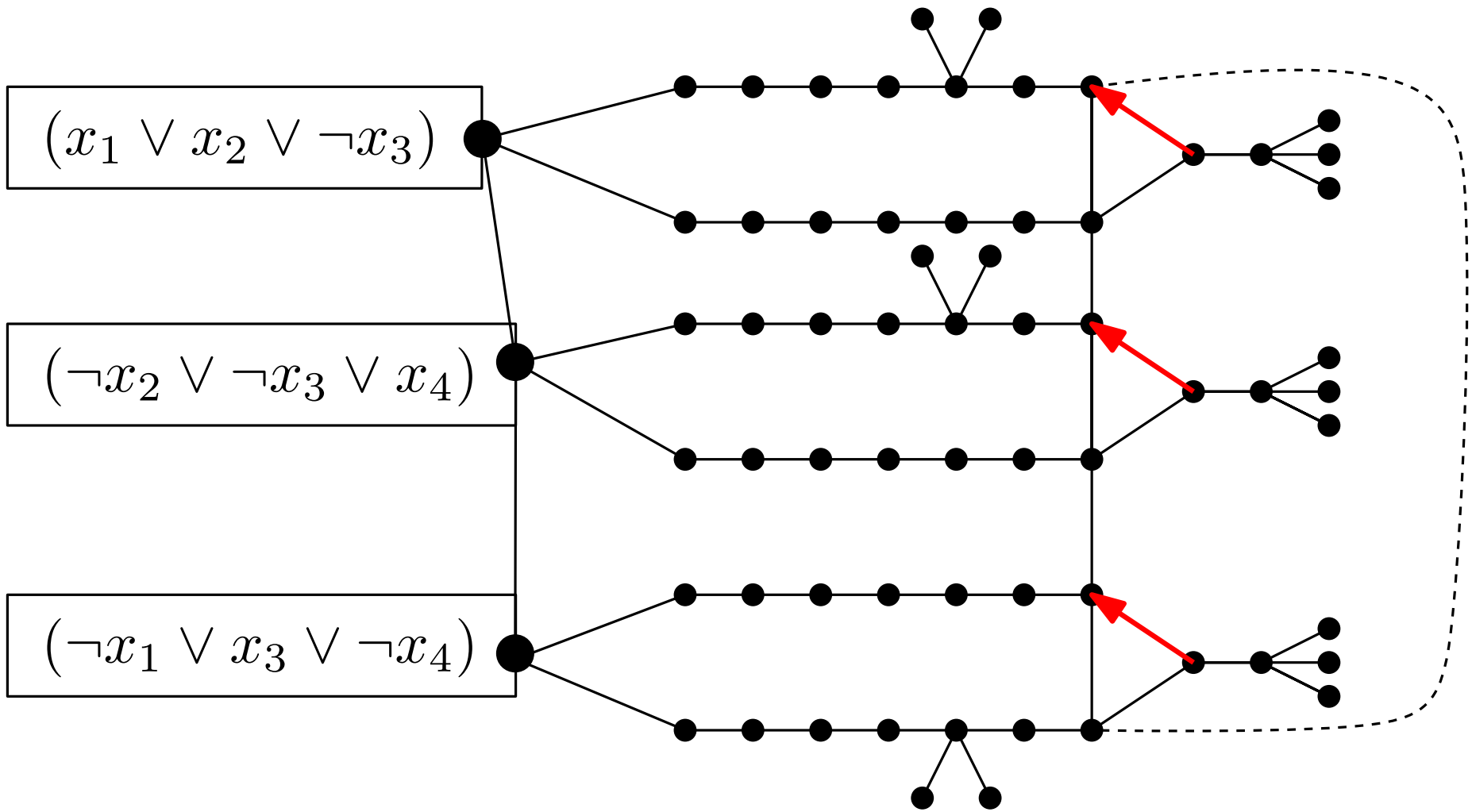

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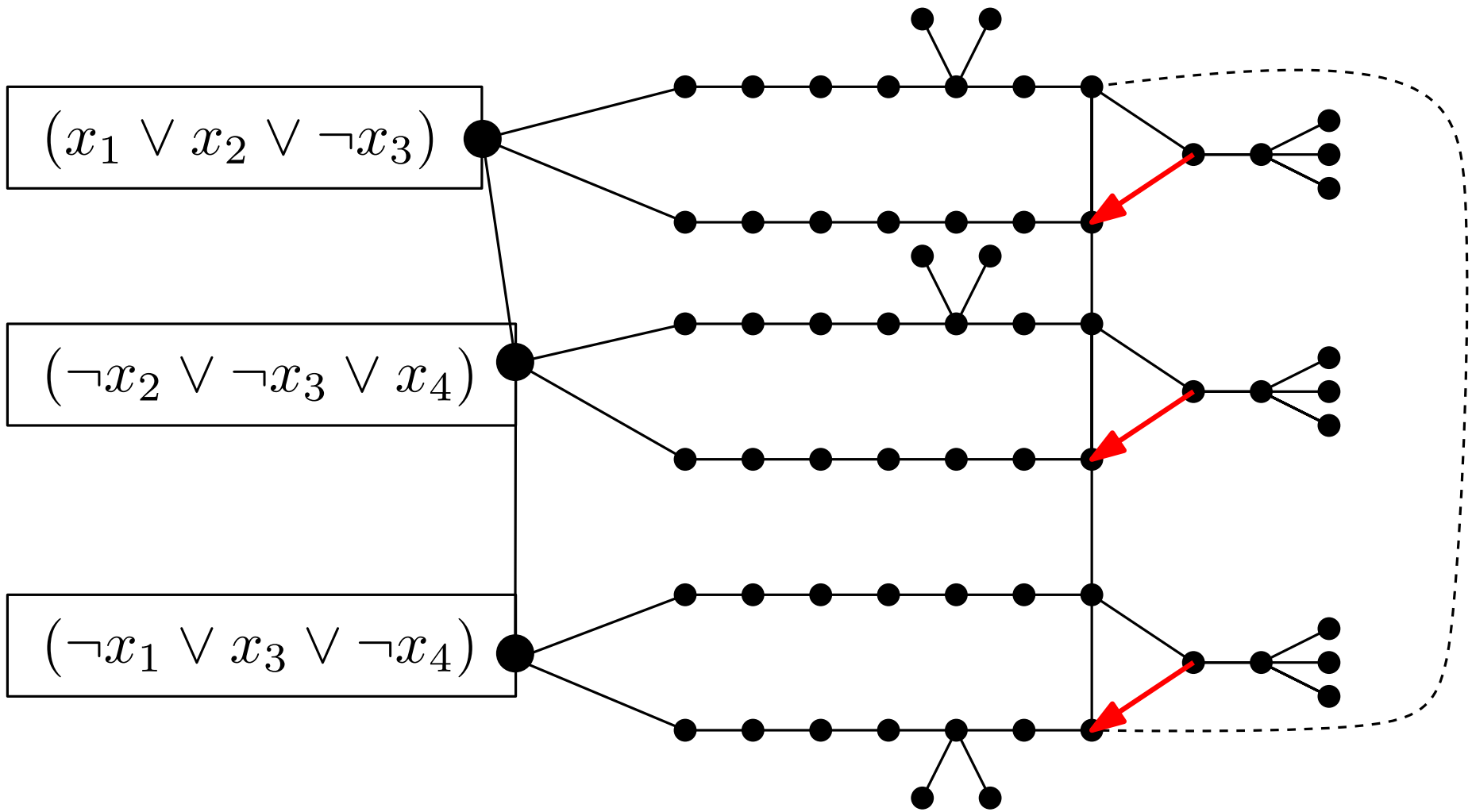

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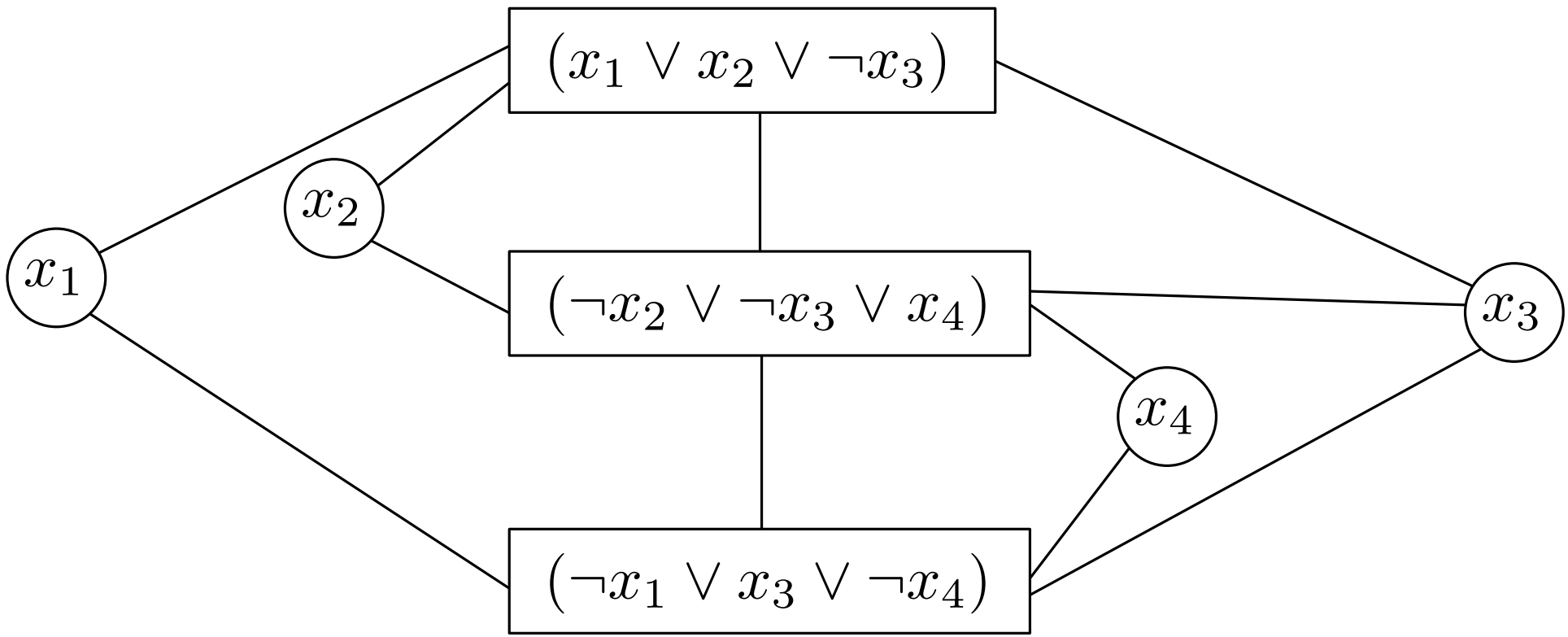

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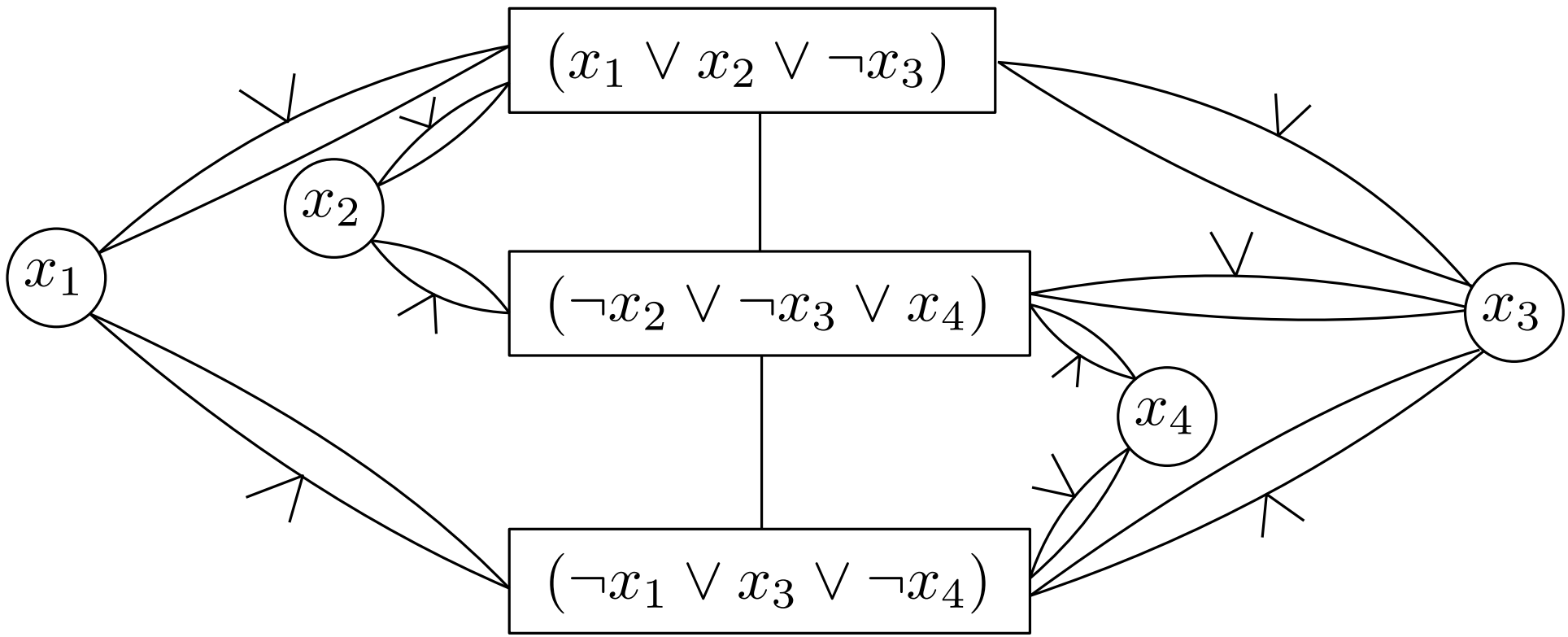

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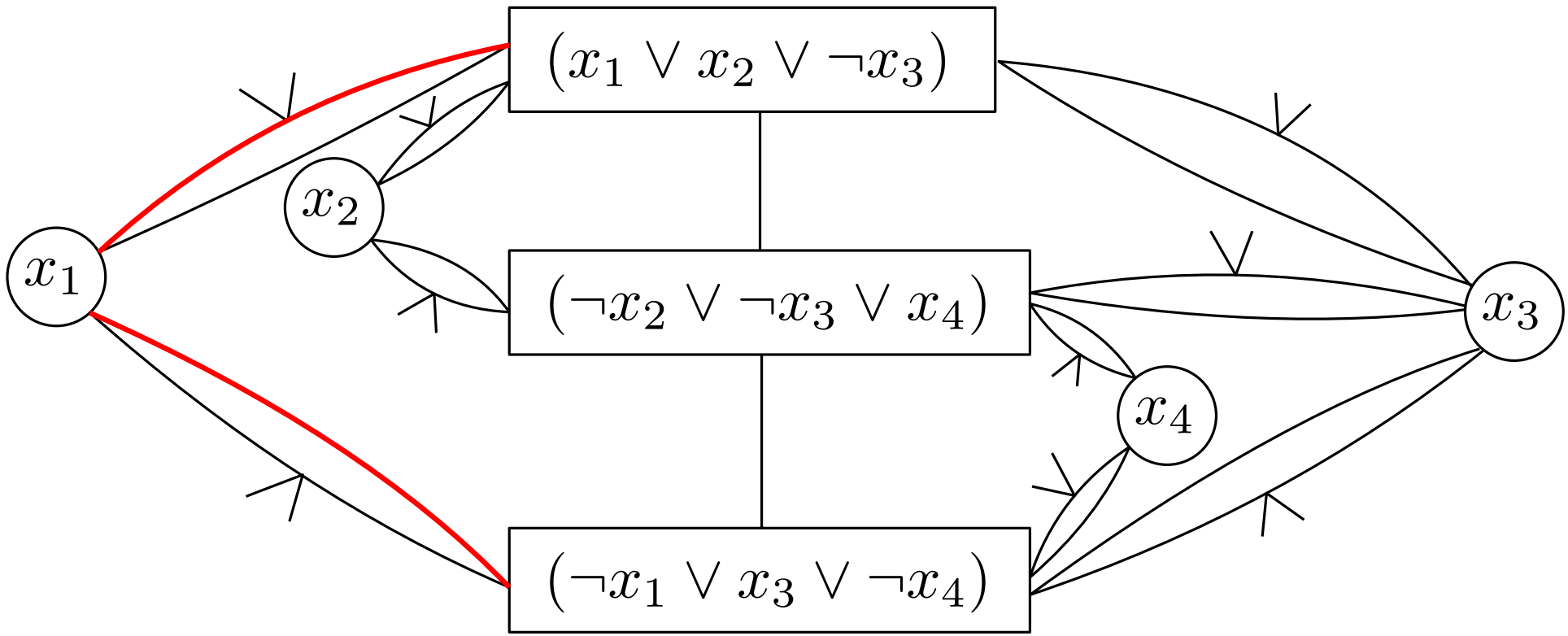

$$x_1 = T$$

The reduction for planar graphs

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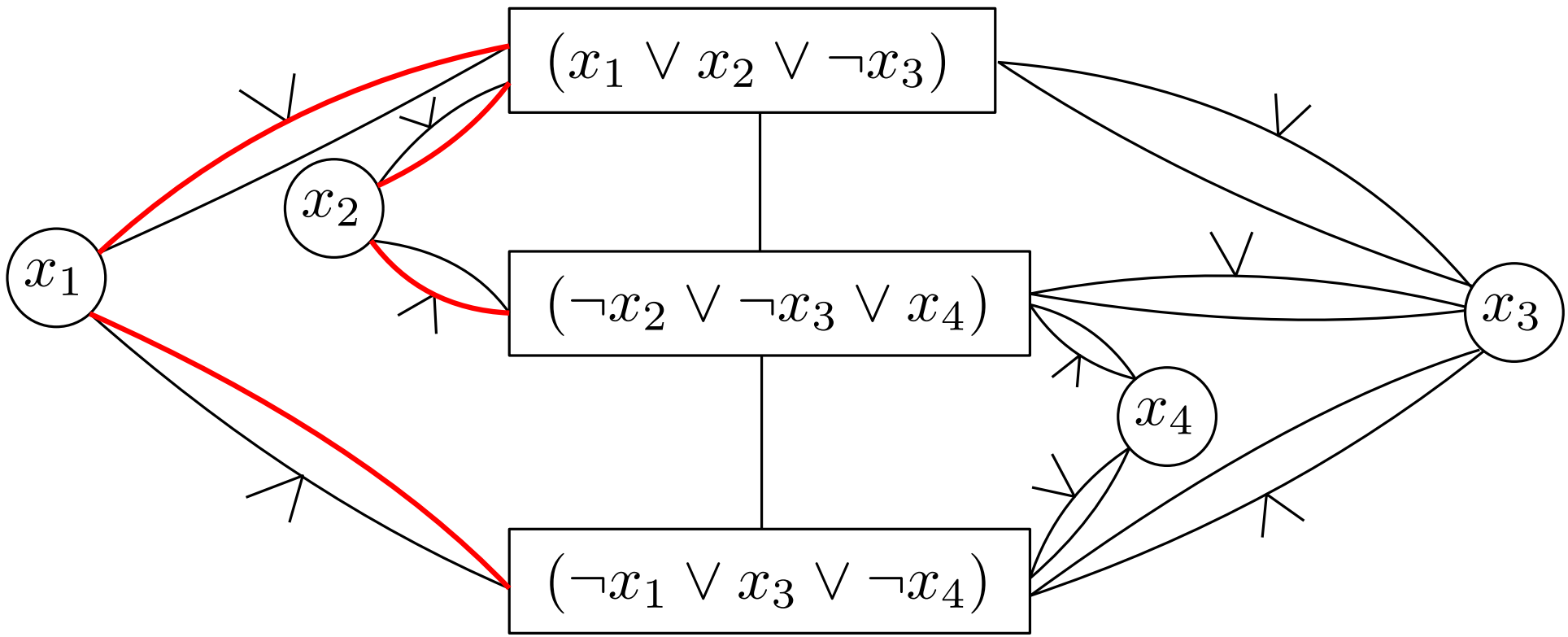

$$x_1 = T \qquad x_2 = F$$

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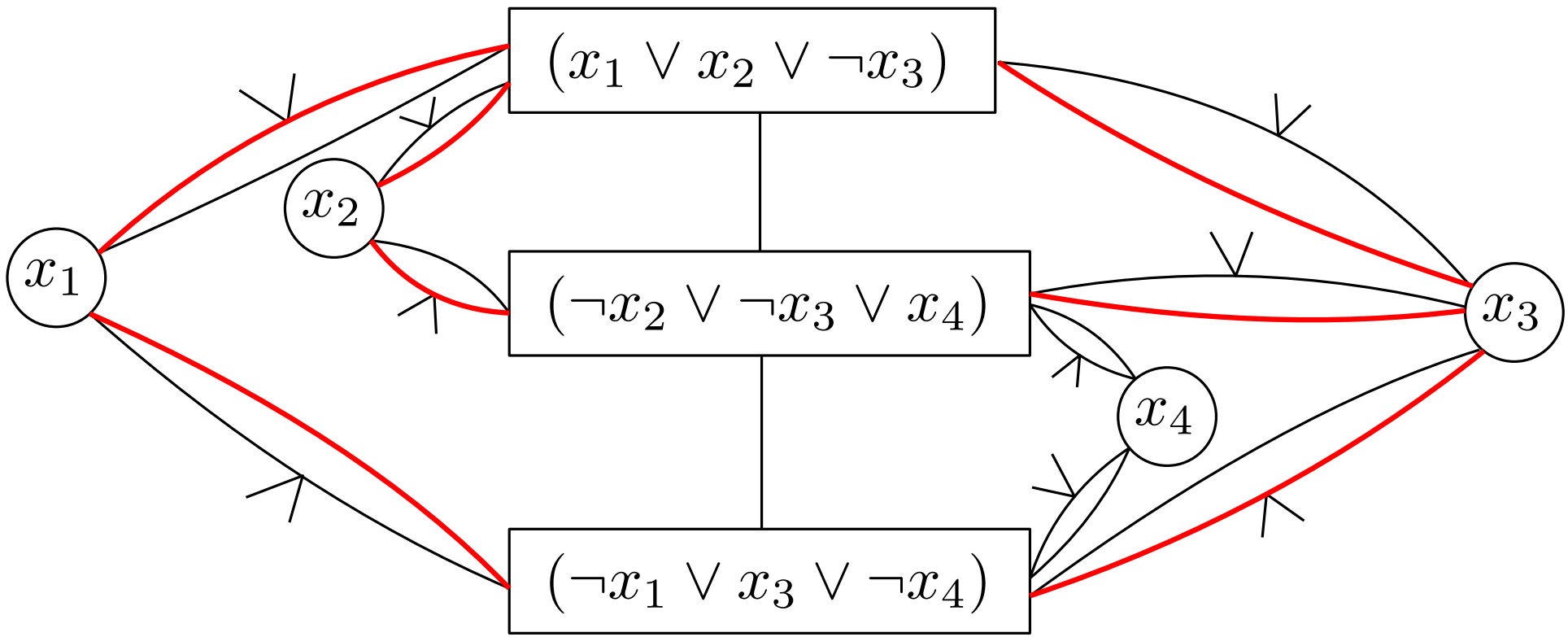

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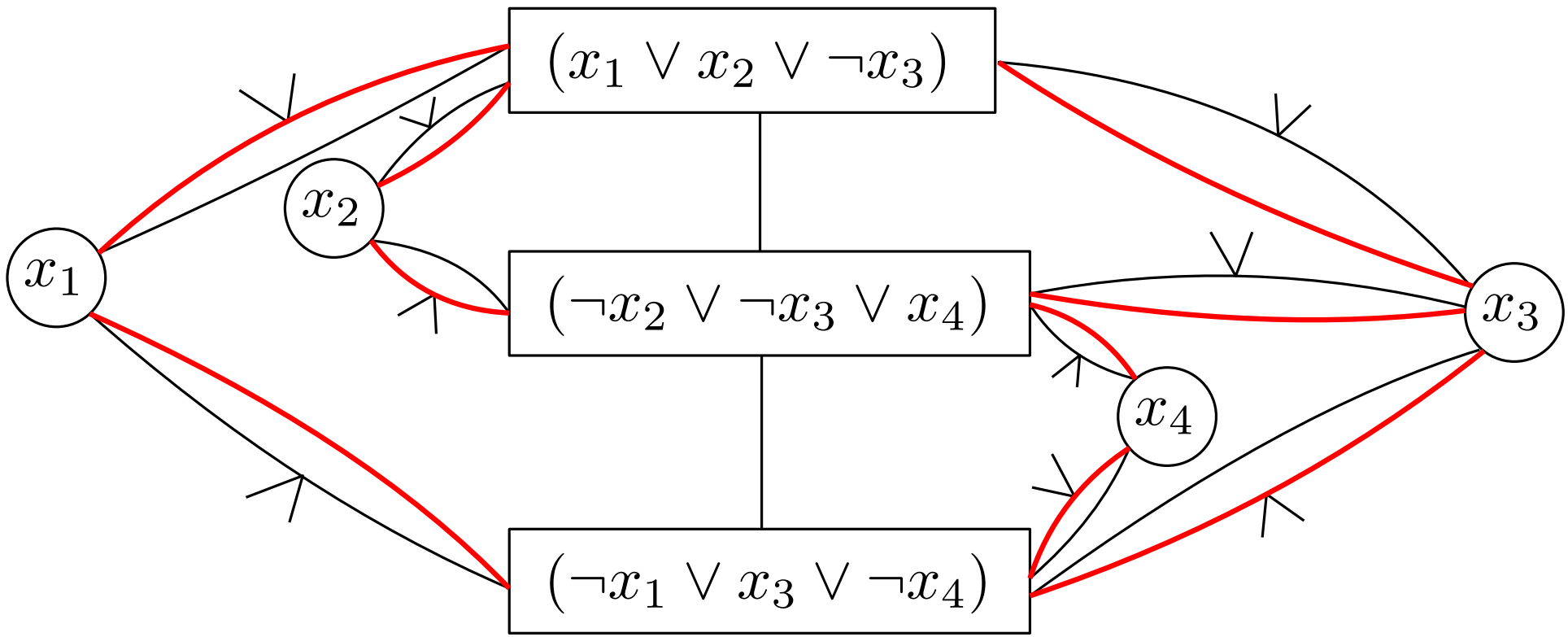

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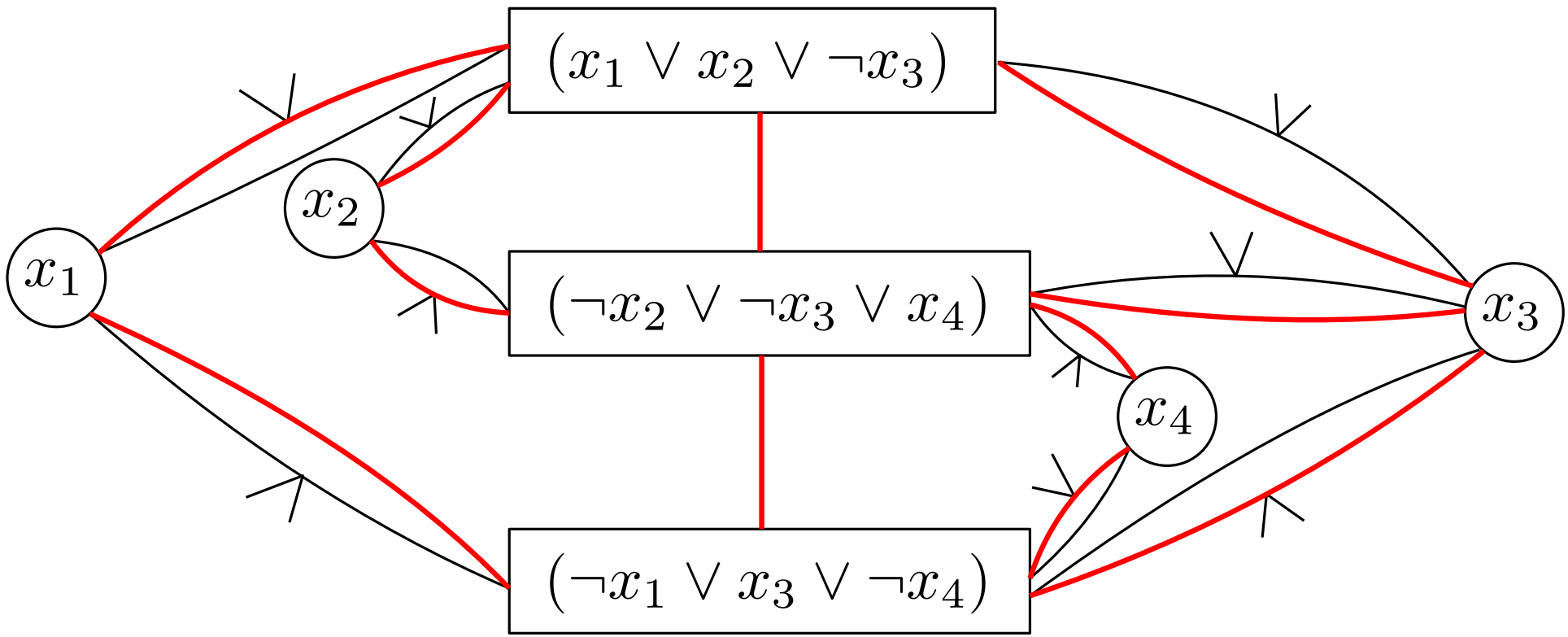

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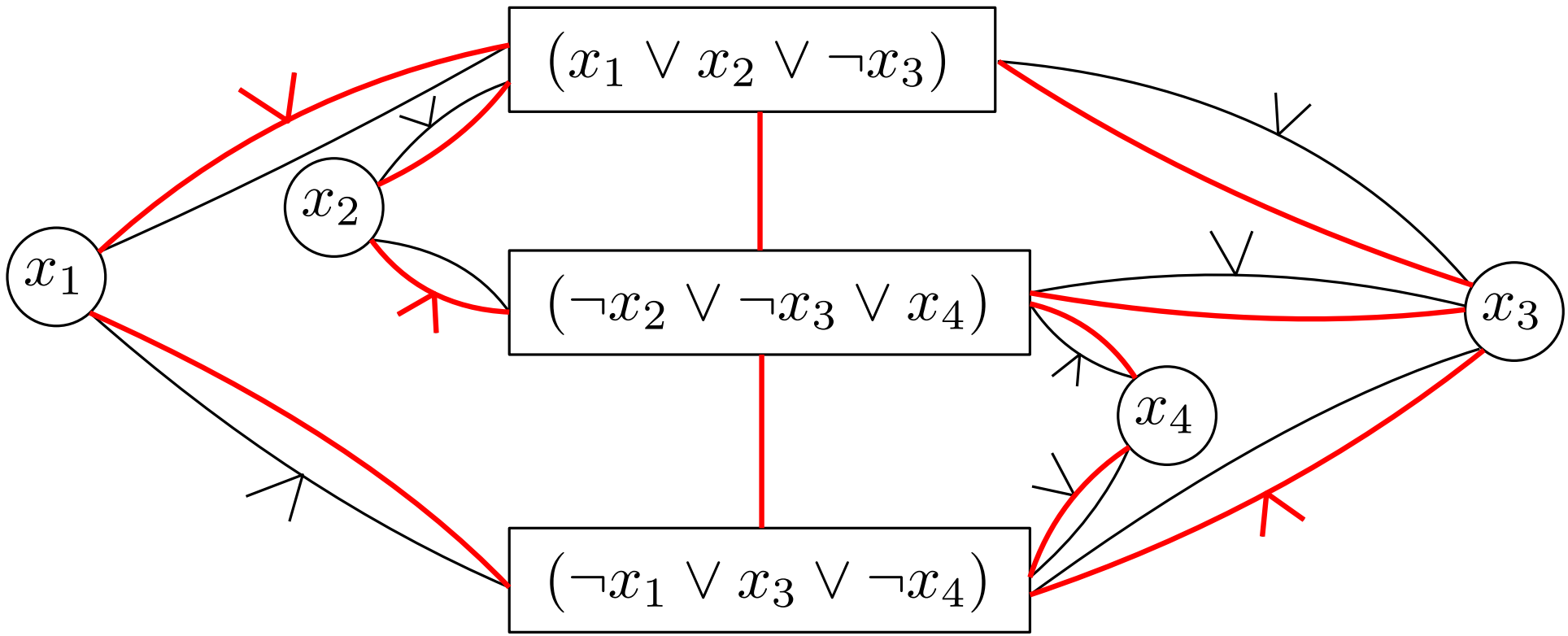

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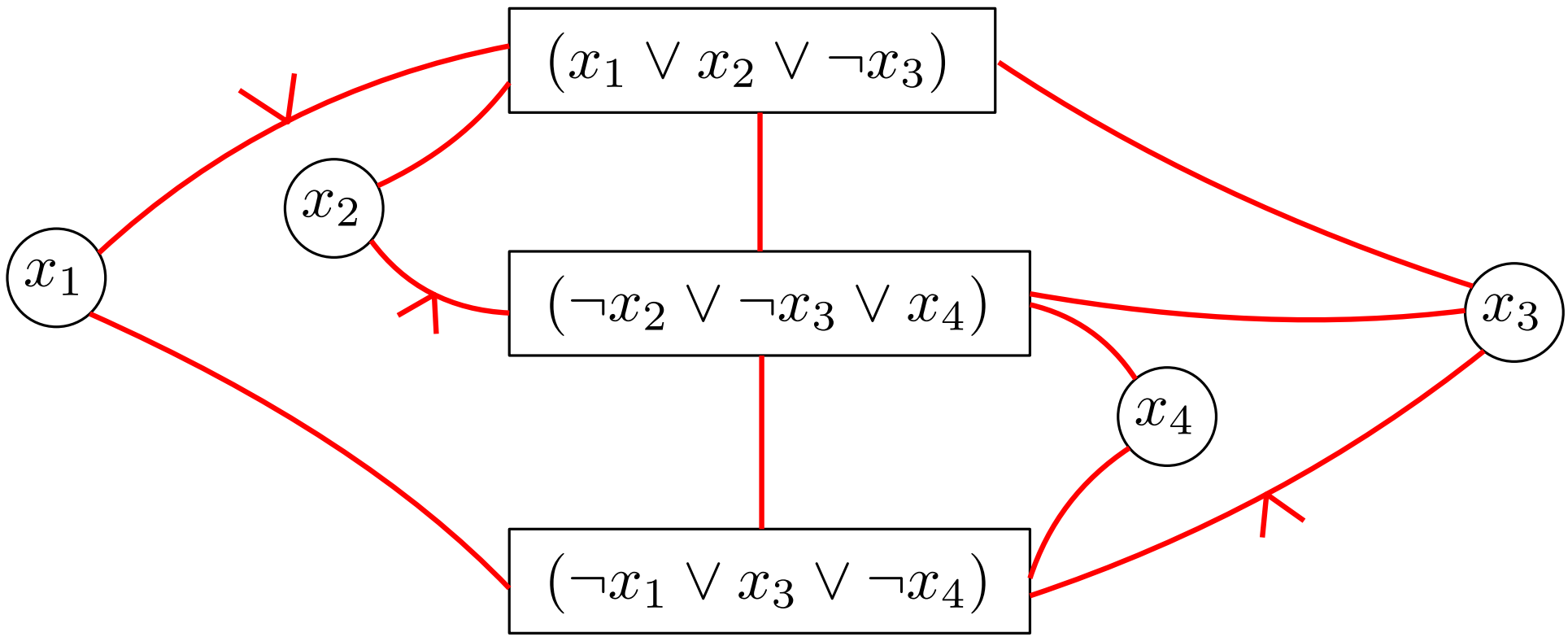

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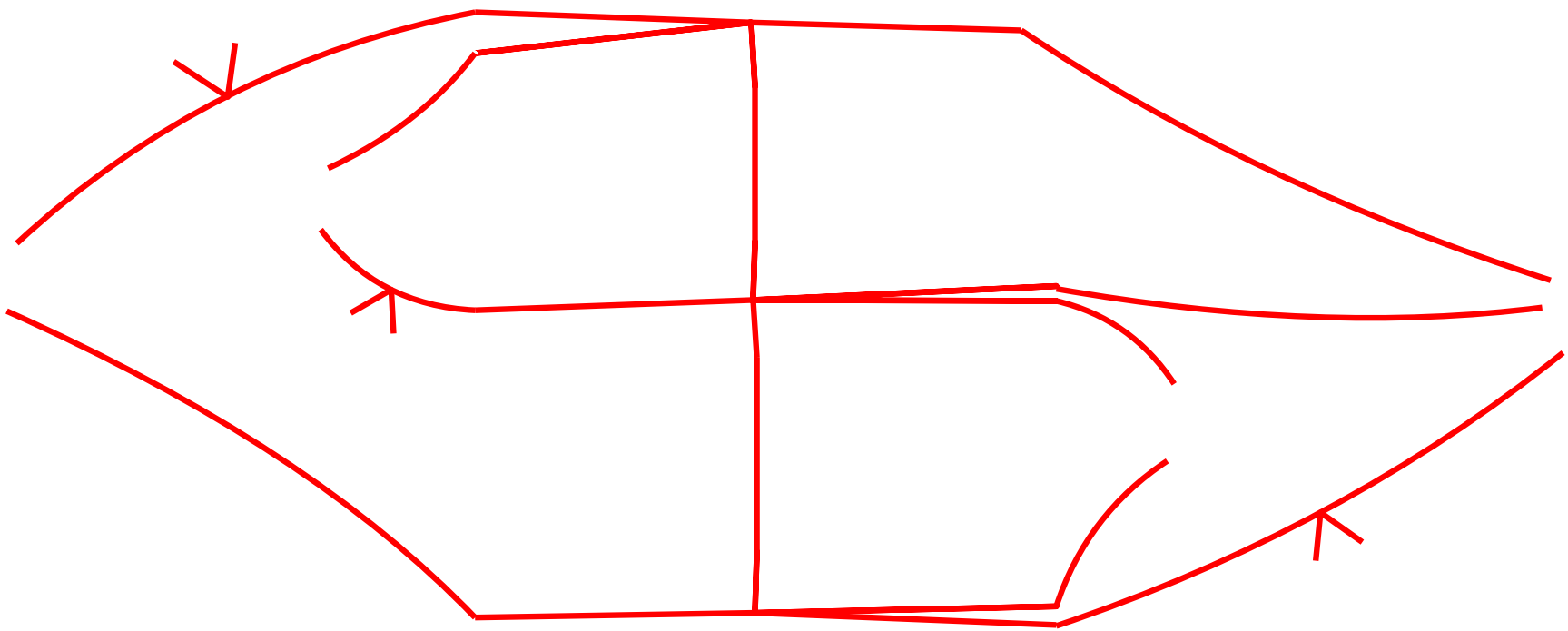

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Thank you!